SAR as a measurement device: data, images and interferometry

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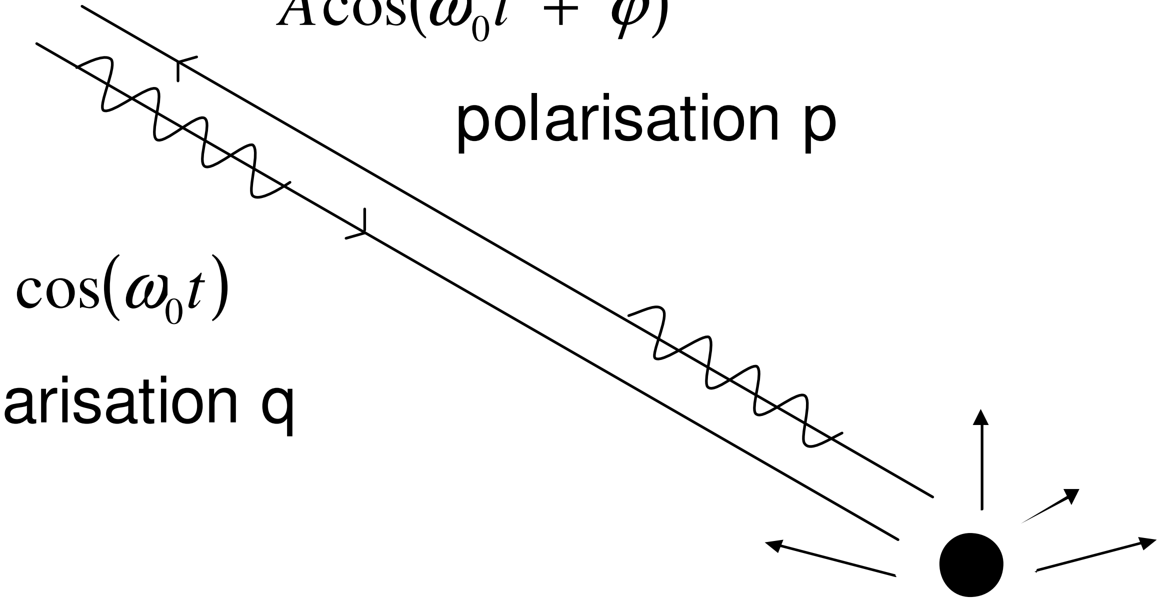
The basic SAR measurement



$A\cos(\omega_0 t + \varphi)$

polarisation p

$\cos(\omega_0 t)$

polarisation q

A SAR measures $S_{pq} = \left(A\cos\varphi, A\sin\varphi\right) \equiv A\,e^{j\varphi}$

This is the **complex image.**

The scattering matrix

A more complete description of a single SAR measurement from a point target is given by :

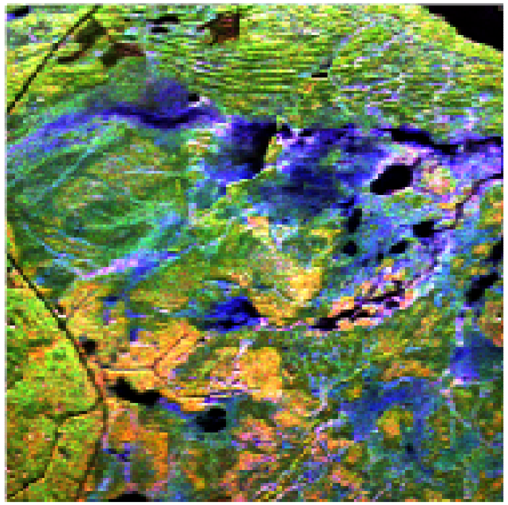
$$\begin{pmatrix} E_p^s \\ E_q^s \end{pmatrix} = \frac{e^{2\pi i R/\lambda}}{R} \begin{pmatrix} S_{pp} & S_{pq} \\ S_{qp} & S_{qq} \end{pmatrix} \begin{pmatrix} E_p^i \\ E_q^i \end{pmatrix}$$

where p and q are polarisations.
Here *R* is the range from the sensor to the target and λ is the wavelength.
Reciprocity is normal for natural targets, i.e. , $S_{pq} = S_{qp}$

so we can represent the scattering matrix by a 3-vector:

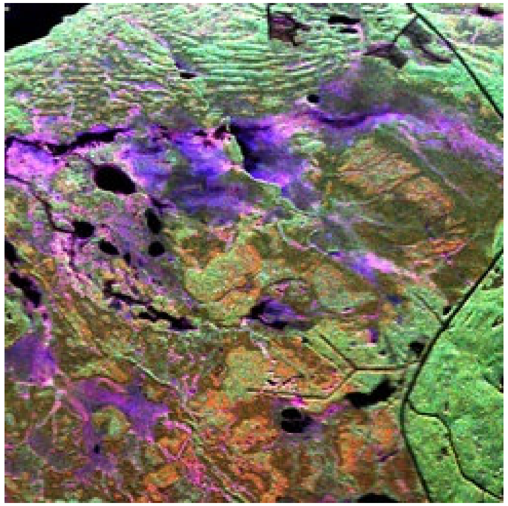
$$\mathbf{S} = \left(S_{pp}, S_{pq}, S_{qq}\right)^t$$

Frequency and Polarisation



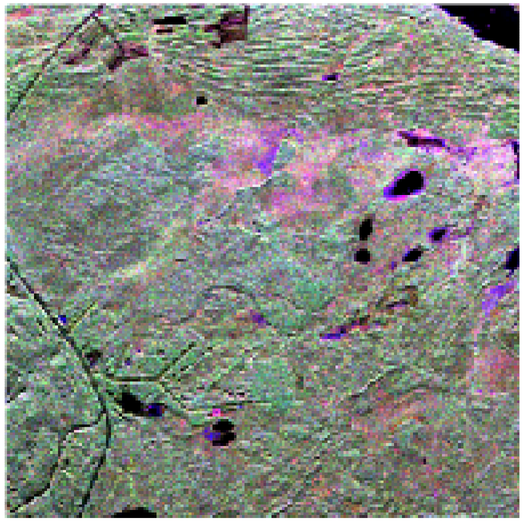
P-band (HH, HV, VV)

λ = 70 cm
None in space



L-band (HH, HV, VV)

λ = 24 cm
ALOS-PALSAR
JERS

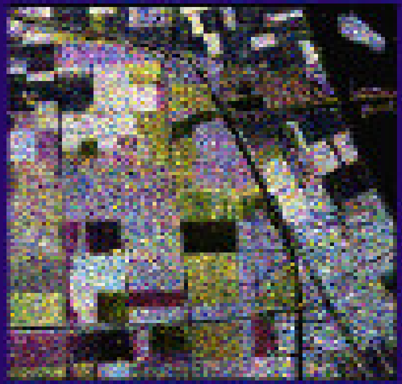


C-band (HH, HV, VV)

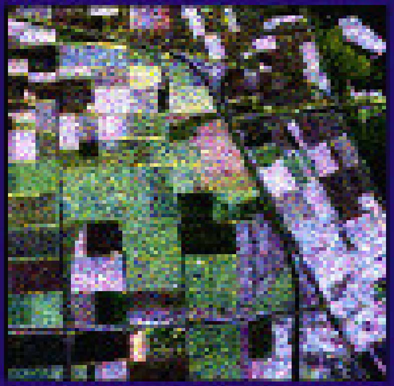
λ = 6 cm
Envisat, Radarsat
ERS

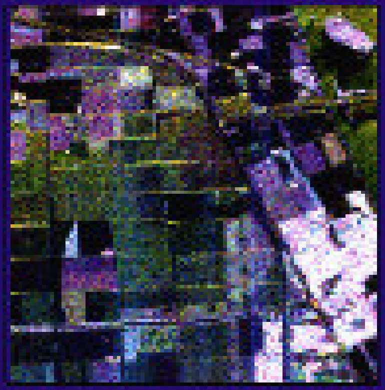
Frequency & Polarisation Comparison

Flevoland, Netherlands **Agricultural Scene**



C-Band





L-Band

P-Band

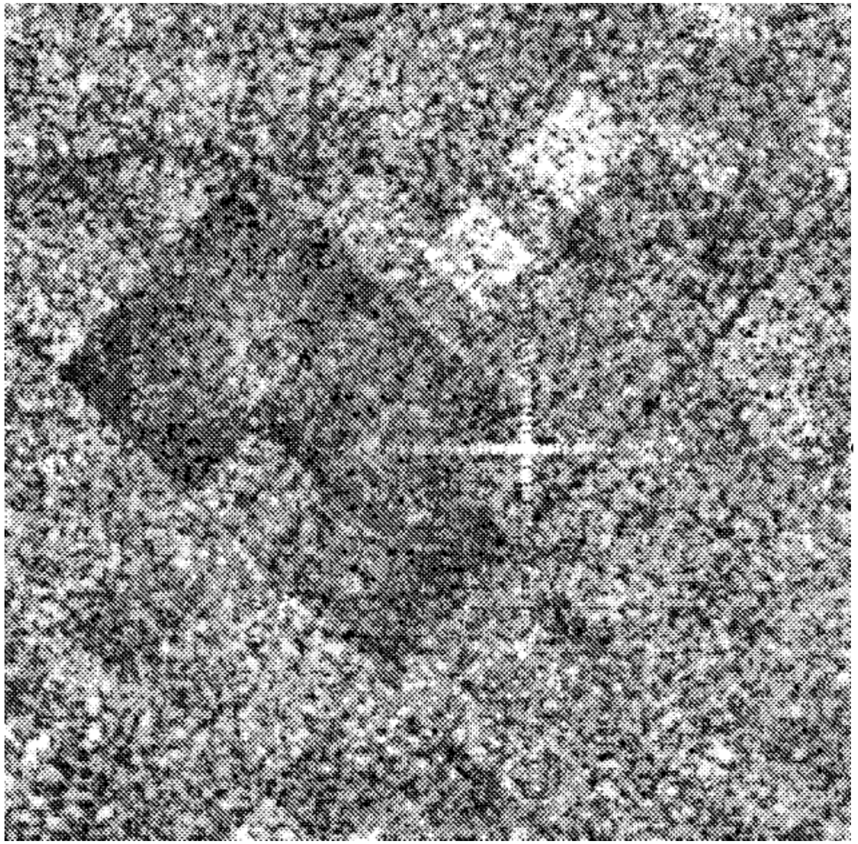
The Radar Cross-Section

The Radar Cross-Section (RCS) of a Point Scatterer is given by

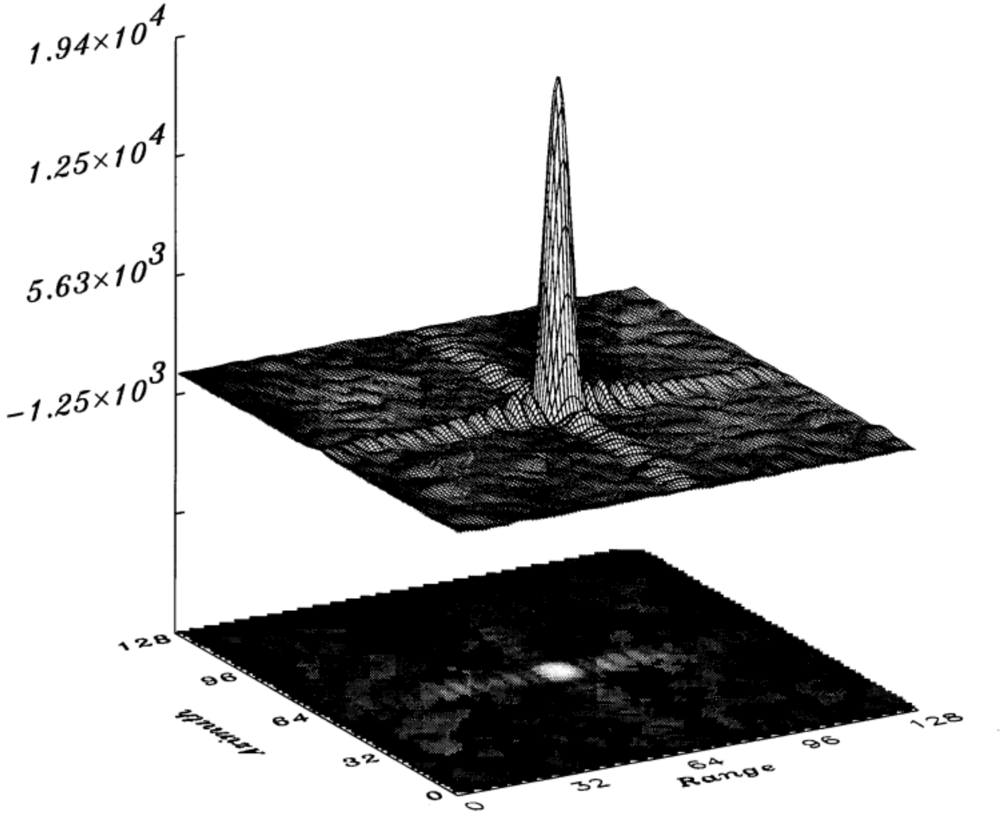
$$\sigma_{pq} = 4\pi \left| S_{pq} \right|^2 = 4\pi R^2 P_s / P_i \qquad [\mathrm{m}^2]$$

where P_i and P_s are the incident and scattered powers (important for calibration).

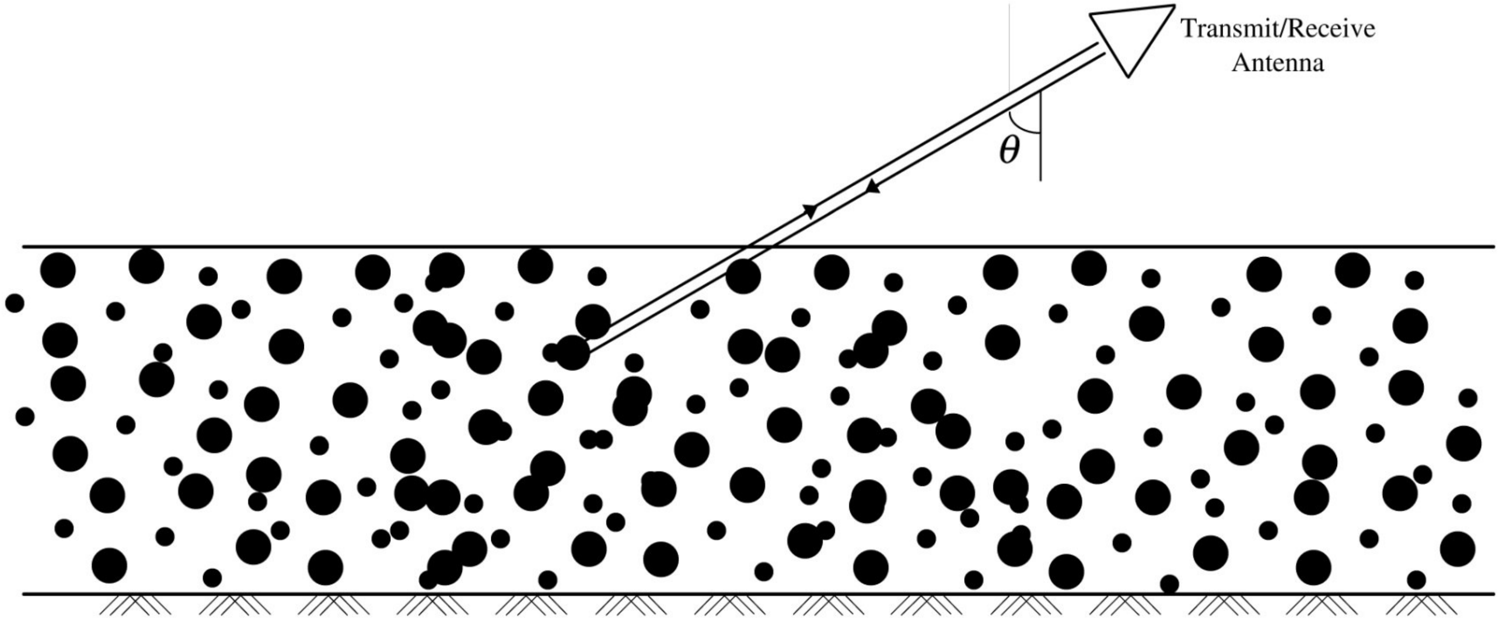
The point spread function



ERS-1 image



Distributed scatterers



Many natural media can be thought of as a collection of randomly positioned point scatterers, each with its own scattering matrix. Now we need a statistical description of the target.

Random vectors and speckle

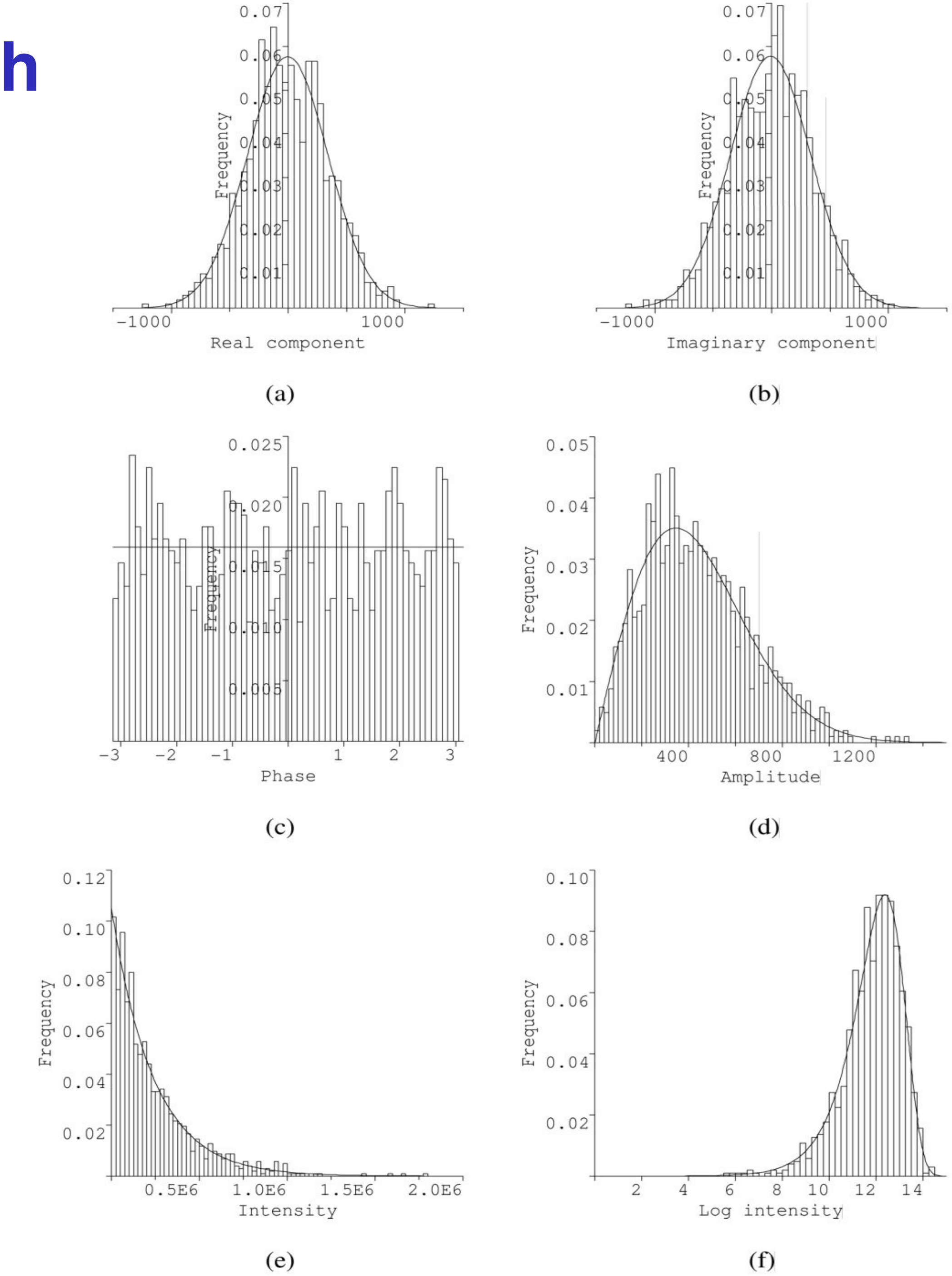
The scattering vector, $\mathbf{S}$, measured at each pixel is now a random vector.

Each complex component, S_{pq}, is “speckled”, i.e. obeys a zero mean complex Gaussian distribution, so:

1. Its real and imaginary parts are zero-mean Gaussian with the same variance.
2. The phase is uniformly distributed between 0 and 2π.
3. Its intensity, $I = |S_{pq}|^2 = \text{real}^2 + \text{imag}^2$, has an exponential distribution, so mean(I) = SD(I).

SAR statistics in each polarisation channel

All these distributions are constructed from a single number, $\sigma = \text{mean}(I)$



(a) (b)

(c) (d)

(e) (f)

Calibrated measurements: the backscattering coefficient

For a single polarisation, the **differential backscattering coefficient**, σ^o, is given by

$$\sigma^o = \frac{4\pi R^2}{\Delta A}\frac{P_s}{P_i} \quad [\mathrm{m}^2/\mathrm{m}^2]$$

where ΔA is the area of a facet of the illuminated surface over which the phase can be considered constant. In practice, it is treated as the area of the SAR pixel when calculating σ^o.

Estimating the backscattering coefficient

Given L independent measurements from a uniform distributed target, the Maximum Likelihood Estimator of σ^0 is given by

$$I = \frac{1}{L}\sum_{k=1}^{L} I^{(k)}$$

where the $I^{(k)}$ are individual intensity measurements.

This does not depend on the original form of the data (amplitude, log, intensity or complex).

L is called the **number of looks**.

The multi-look intensity distribution

The PDF of *I* given *L* looks is a gamma distribution:

$$P_I(I) = \frac{1}{\Gamma(L)}\left(\frac{L}{\sigma}\right)^L I^{L-1} e^{-LI/\sigma}$$

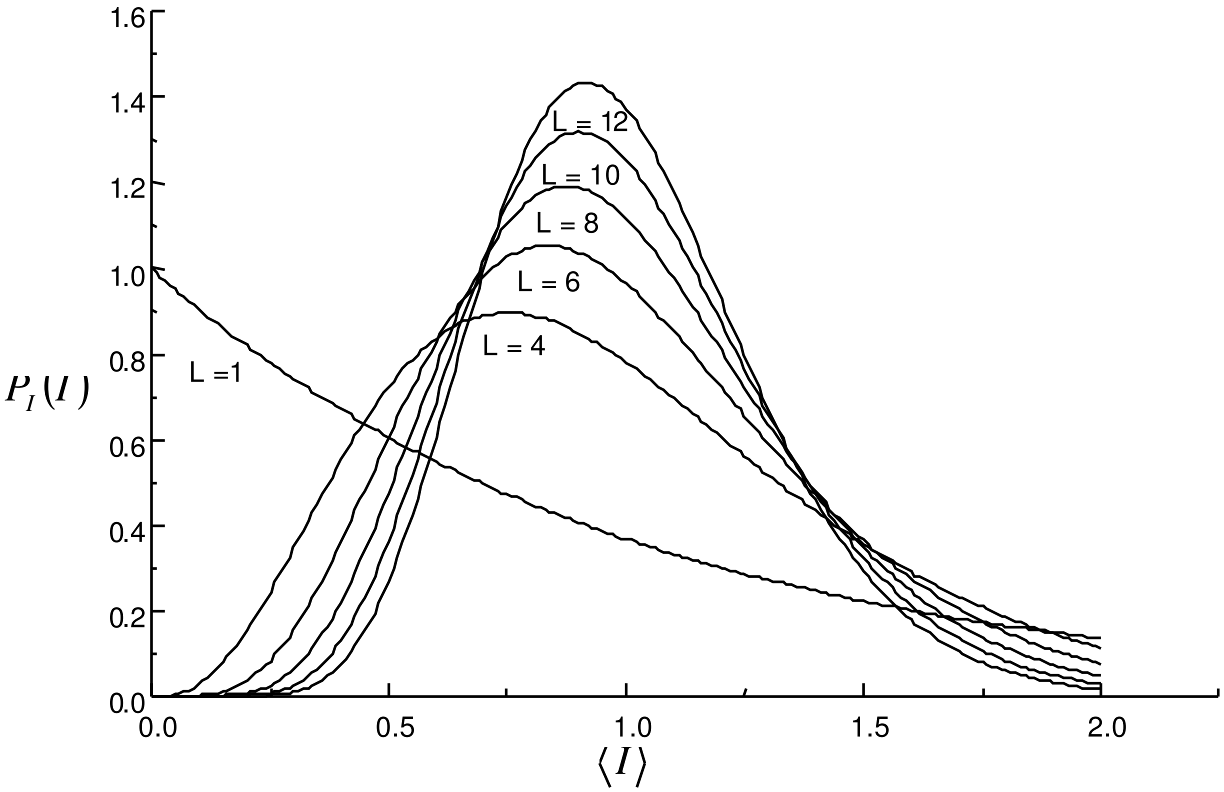
with $\langle I \rangle = \sigma$ $\mathrm{var}(I) = \frac{\sigma^2}{L}$

Coefficient of variation = CV = SD/ mean = $\frac{1}{\sqrt{L}}$.

Putting *L* = 1 (single look) gives SD = mean, CV = 1.

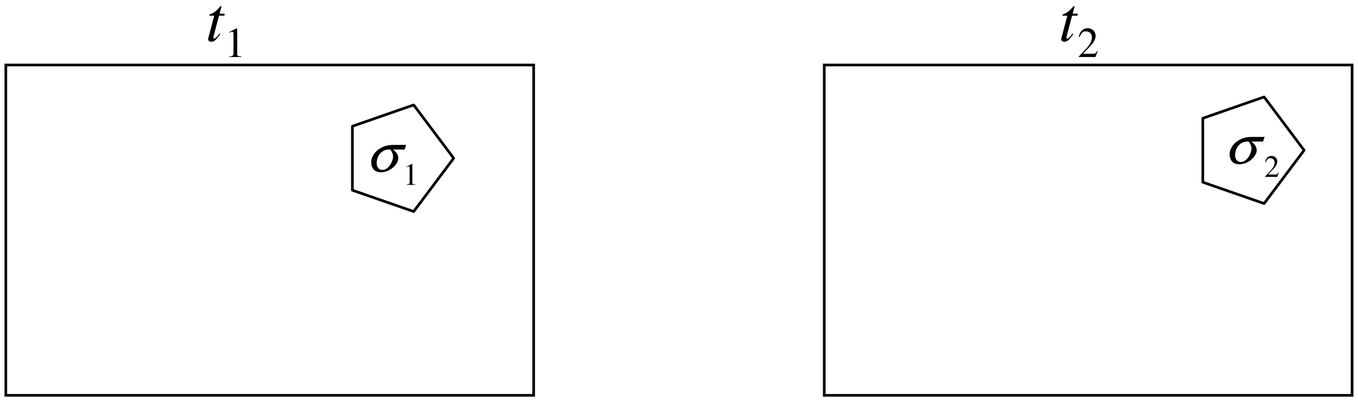
ENL = Equivalent number of looks = $(\text{mean})^2$/variance

The gamma distribution

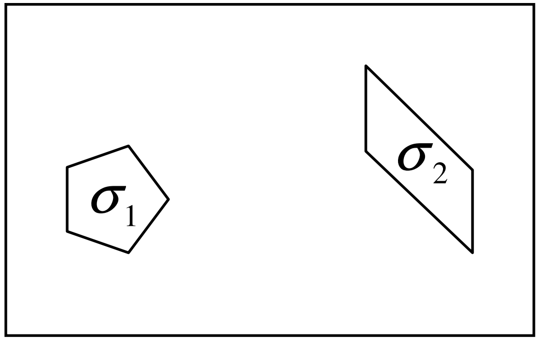


Unit mean Gamma distributions of orders of 1, 4, 6, 8, 10 and 12. The distribution tends to normality as *L* increases.

Changes and differences in backscatter



Temporal changes



Spatial differences

Change based on ratios

The distribution of the ratio $Z = I_1 / I_2$ is given by

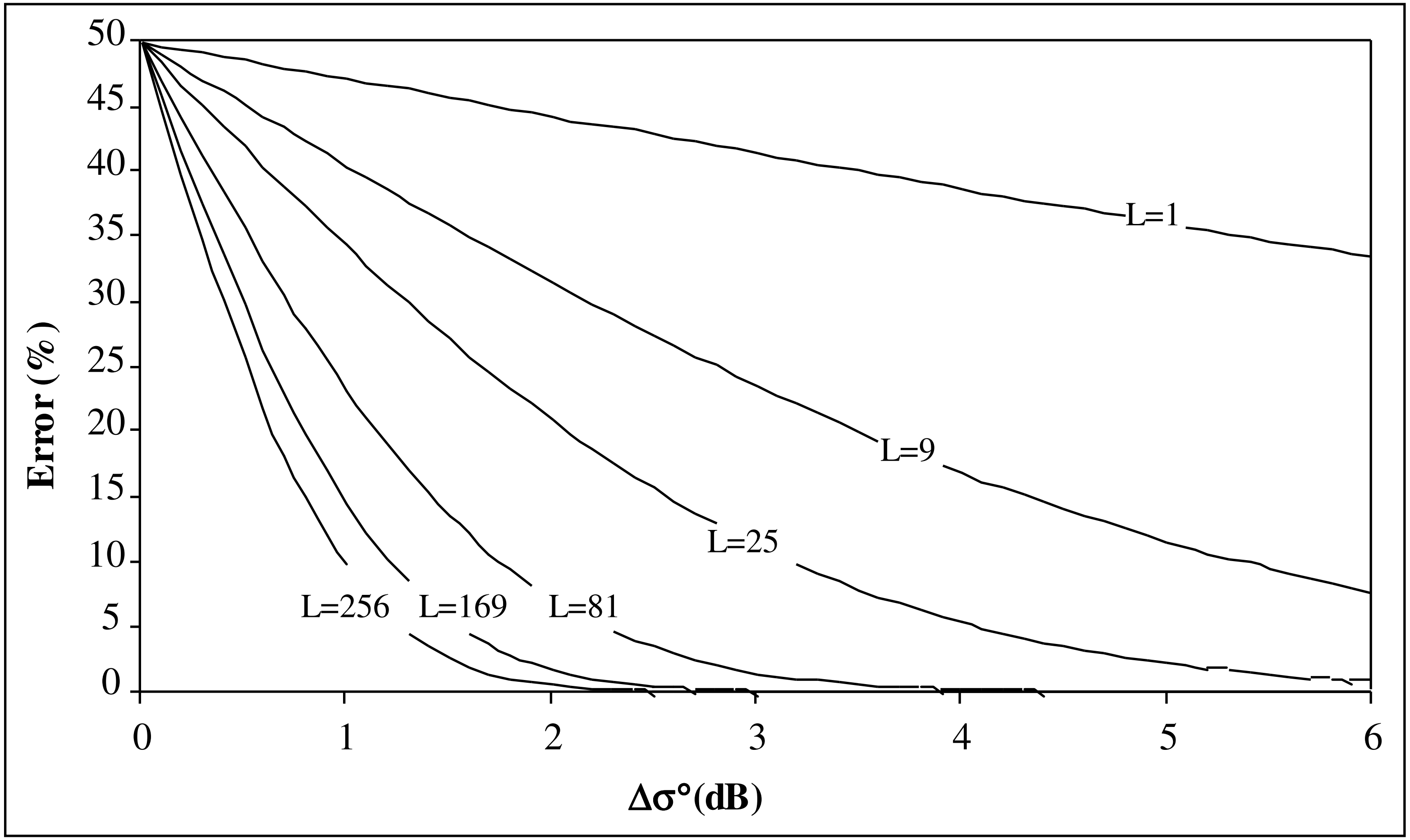
$$p(Z) = \frac{\Gamma(2L)\gamma^L Z^{L-1}}{\Gamma^2(L)(\gamma + Z)^{2L}}$$

where $\gamma = \frac{\sigma_1}{\sigma_2}$ is the true intensity ratio.

Ratio of intensities is equivalent to the difference of logs.

Depends only on the **relative change** in intensity between the images.
Minimises topographic and other multiplicative effects, e.g., calibration errors.

Error in measuring temporal change



Bruniquel. 1996

Information in polarimetric data

In polarimetric data, we have to consider information in the individual channels and in combinations of channels. The information-bearing quantities are of the form

$$C_{pq} = \left\langle S_p S_q^* \right\rangle = \left\langle \left| S_p S_q \right| \exp\{j(\phi_p - \phi_q)\} \right\rangle$$

This is a complex covariance containing an amplitude and phase difference term.

When p=q,

$$C_{pp} = \sigma_p = \left\langle S_p S_p^* \right\rangle = \left\langle \left| S_p \right|^2 \right\rangle = \left\langle I_p \right\rangle = \text{mean intensity}$$

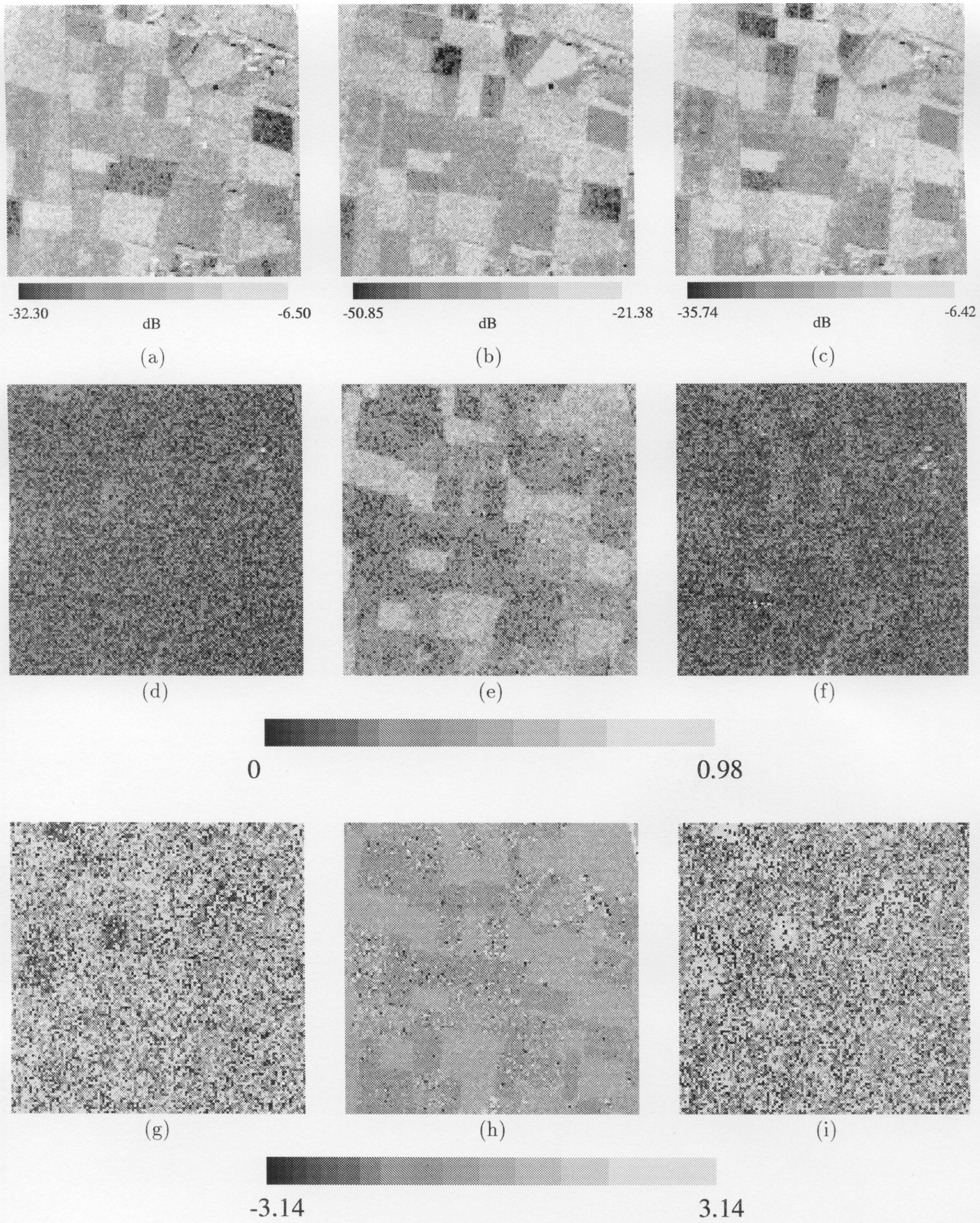
If the data are calibrated, the σ_p are backscattering coefficients.

The covariance matrix

We can say something even stronger:

homogeneous distributed targets are completely characterised by a 3-dimensional Gaussian distribution, which is completely determined by its covariance matrix:

$$C = \begin{pmatrix} \sigma_1 & \sqrt{\sigma_1 \sigma_1}\rho_{12} & \sqrt{\sigma_1 \sigma_3}\rho_{13} \\ \sqrt{\sigma_1 \sigma_2}\rho_{12}^* & \sigma_2 & \sqrt{\sigma_2 \sigma_3}\rho_{23} \\ \sqrt{\sigma_1 \sigma_3}\rho_{13}^* & \sqrt{\sigma_2 \sigma_3}\rho_{23}^* & \sigma_3 \end{pmatrix}$$


-32.30
dB
-6.50
(a)
-50.85
dB
-21.38
(b)
-35.74
dB
-6.42
(c)
(d)
(e)
(f)
0
0.98
(g)
(h)
(i)
-3.14
3.14

The complex correlation coefficient

Crucially important for polarimetry and interferometry is the complex correlation coefficient of channel *i* and *j:*

$$\rho_{ij} = \frac{\left\langle S_i \, S_j^* \right\rangle}{\sqrt{\left|S_i\right|^2 \left|S_j\right|^2}}$$

Its magnitude, $\gamma_{ij} = \left|\rho_{ij}\right|$, is known as the coherence.

In an interferometric context, its phase is the interferometric phase.

Estimating coherence

For *N* independent pixels, the MLE of ρ is

$$\frac{\frac{1}{N}\sum_{k=1}^{N} S_1^{(k)} S_2^{(k)*}}{\sqrt{\frac{1}{N}\sum_{1}^{N}\left|S_1^{(k)}\right|^{\hat{\rho}=2} \frac{1}{N}\sum_{1}^{N}\left|S_2^{(k)}\right|^2}}$$

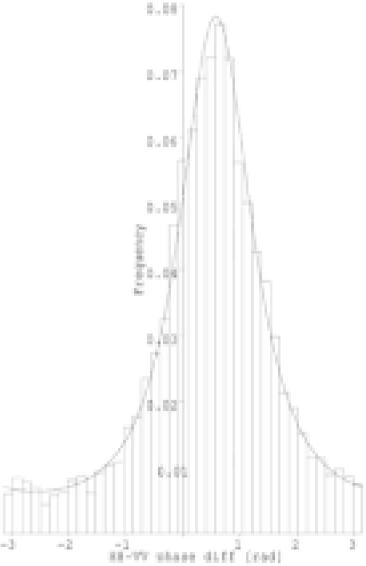
and the MLE of γ is

$$\hat{\gamma} = |\hat{\rho}|$$

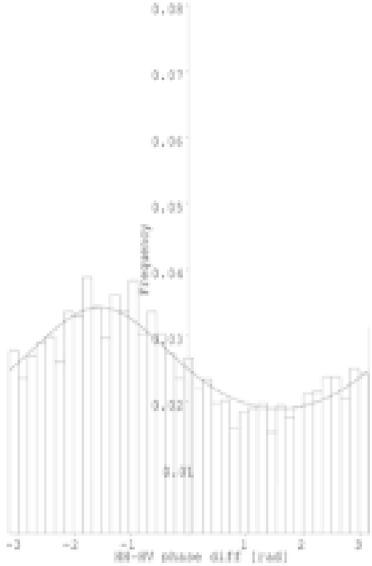
This is the **multi-look** coherence estimate.

(a)

(b)


0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
Frequency
-3
-2
-1
1
2
3
HH-VV phase diff [rad]

(c)


0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
Frequency
-3
-2
-1
1
2
3
HH-VV phase diff [rad]

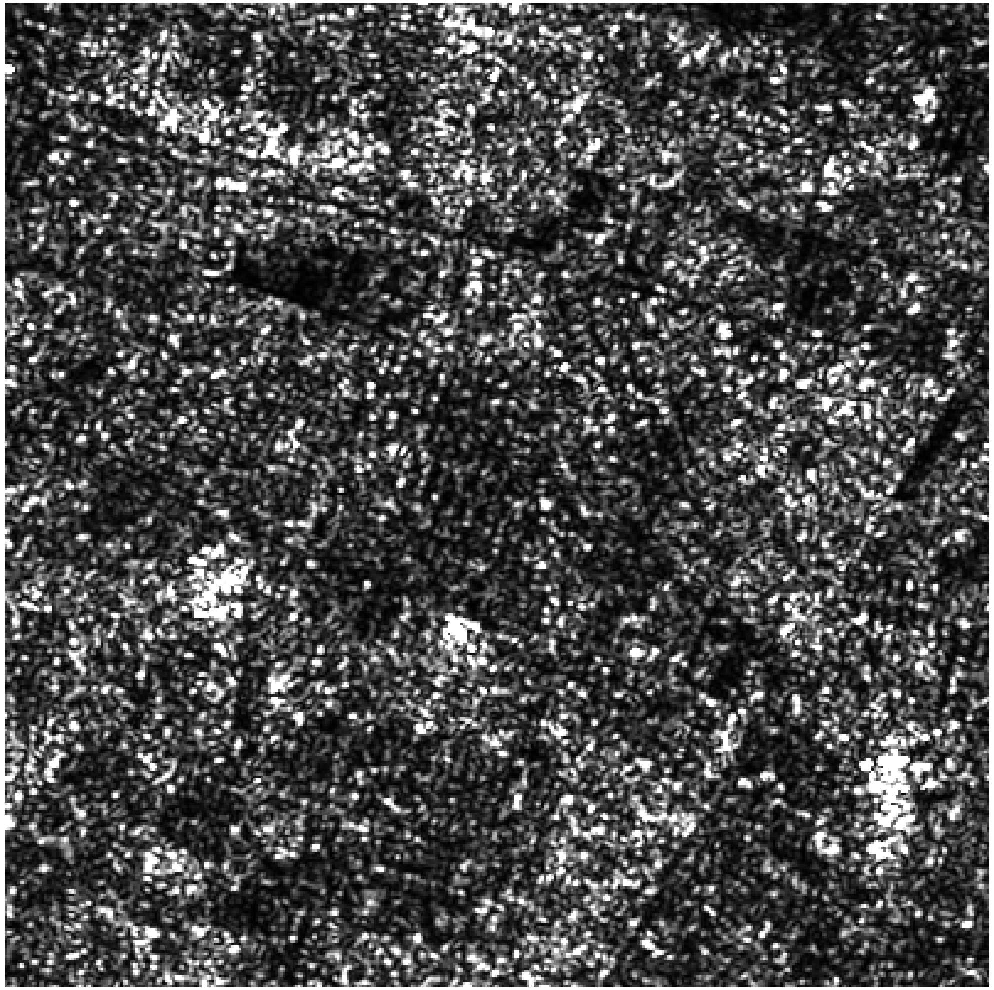
(d)

Multi-channel speckle filtering

Speckle reduction is a key step in many land applications

The usual approach uses spatial filtering applied to individual images – this loses information.

A multi-channel filtering method [Quegan and Yu, 2001] minimises speckle while preserving the radiometry and spatial resolution of the individual channels.

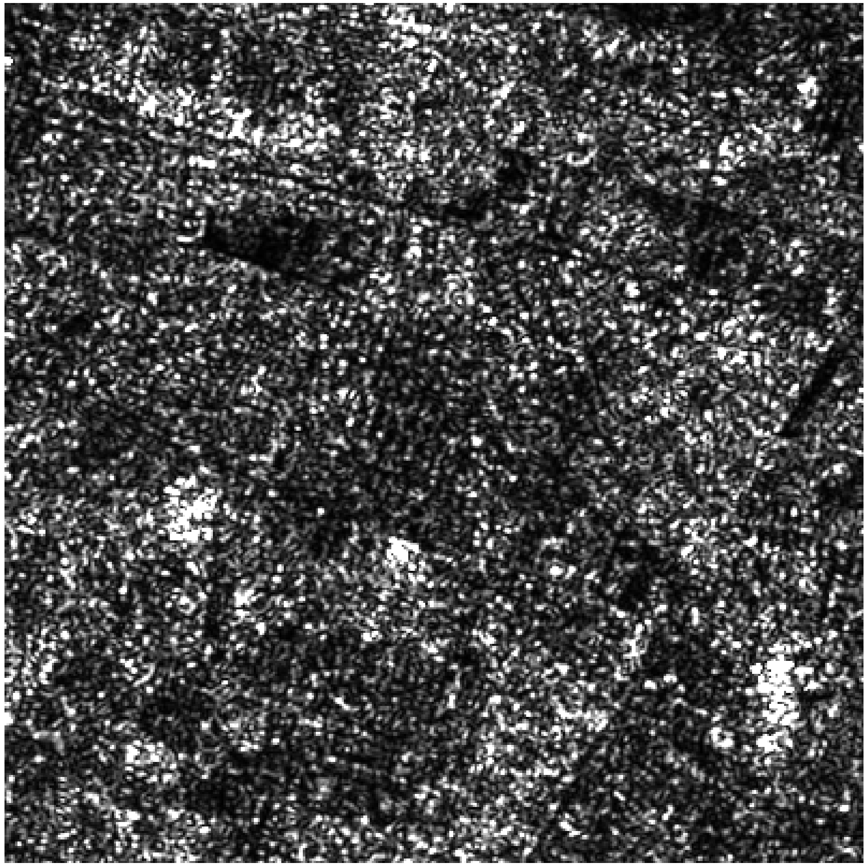


Envisat APP HH image

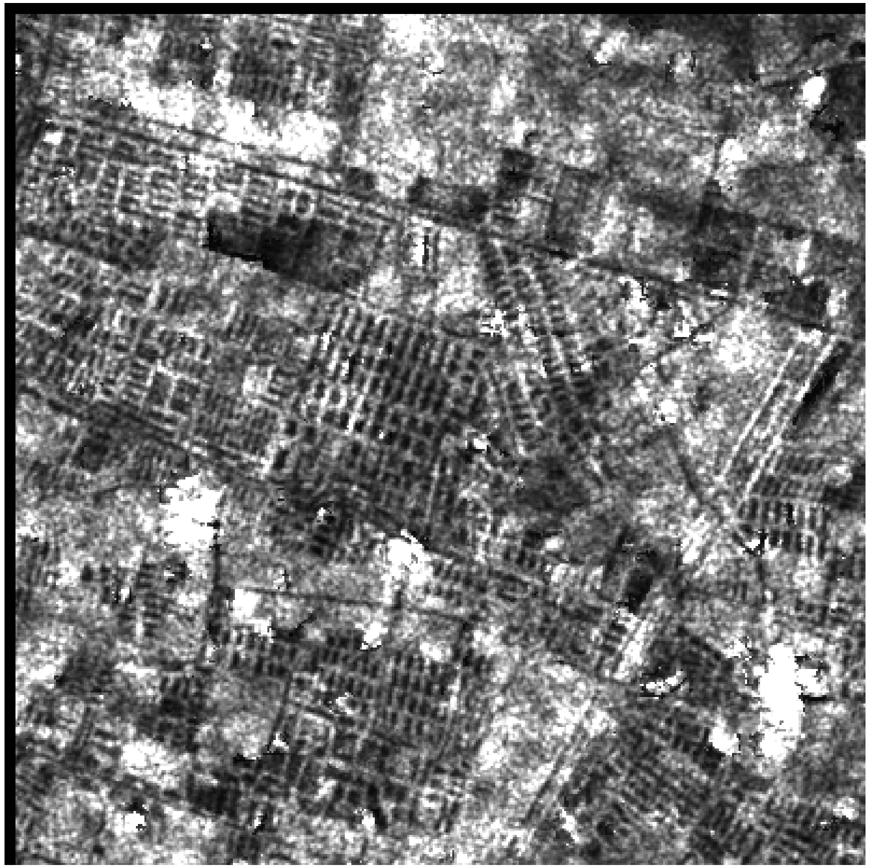
400 x 400 pixels (12.5 x 12.5m)

Gaoyou, Jiangsu province

2004 05 24



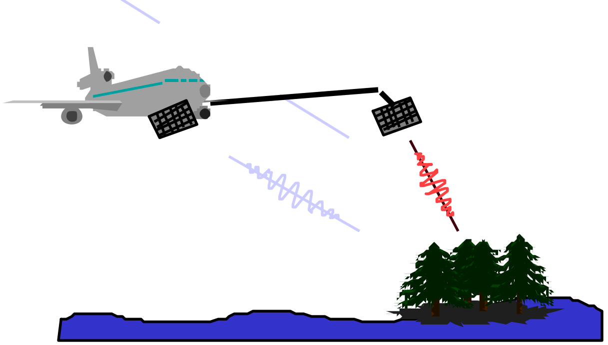
Before filtering



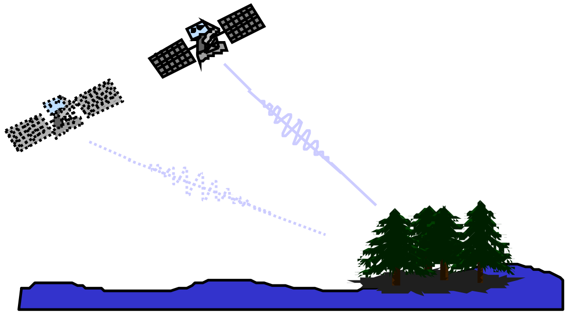
After filtering using10 images
(5 dates, 2 polarisations)

SAR Interferometry

Method: correlate radar signals of the scene taken from two slightly different angles.



Two antennas on one platform



A: Two platforms

B: Repeat orbits of one platform

Single pass interferometry, e.g. SRTM.
Allows height measurement

A: Allows height measurement

B: Crucially dependent on the stability of the dominant scatterers; if STABLE can recover height; if UNSTABLE, can infer biomass using coherence.

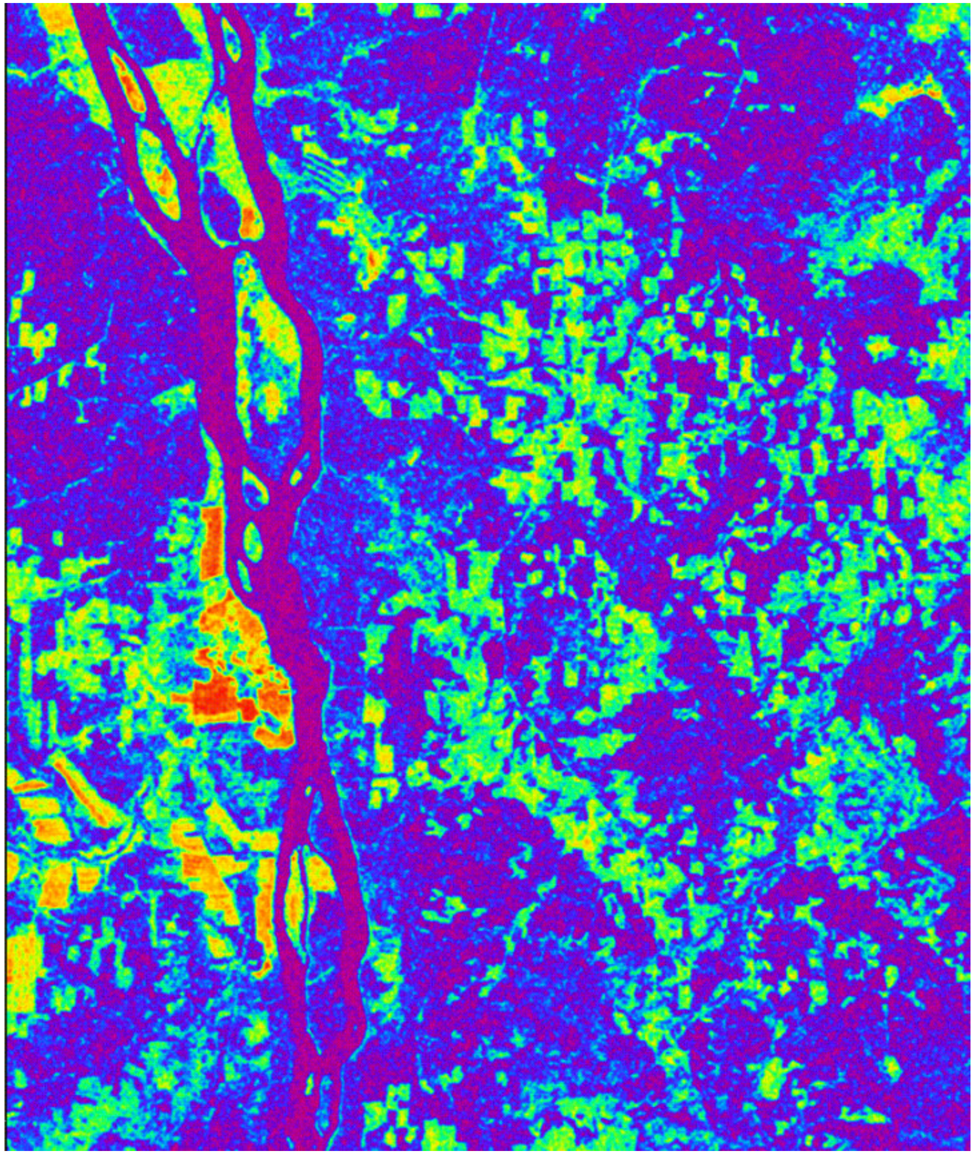
ERS 1/2: 1 day repeat;

RADARSAT: 24 days;

ASAR: 35 days;

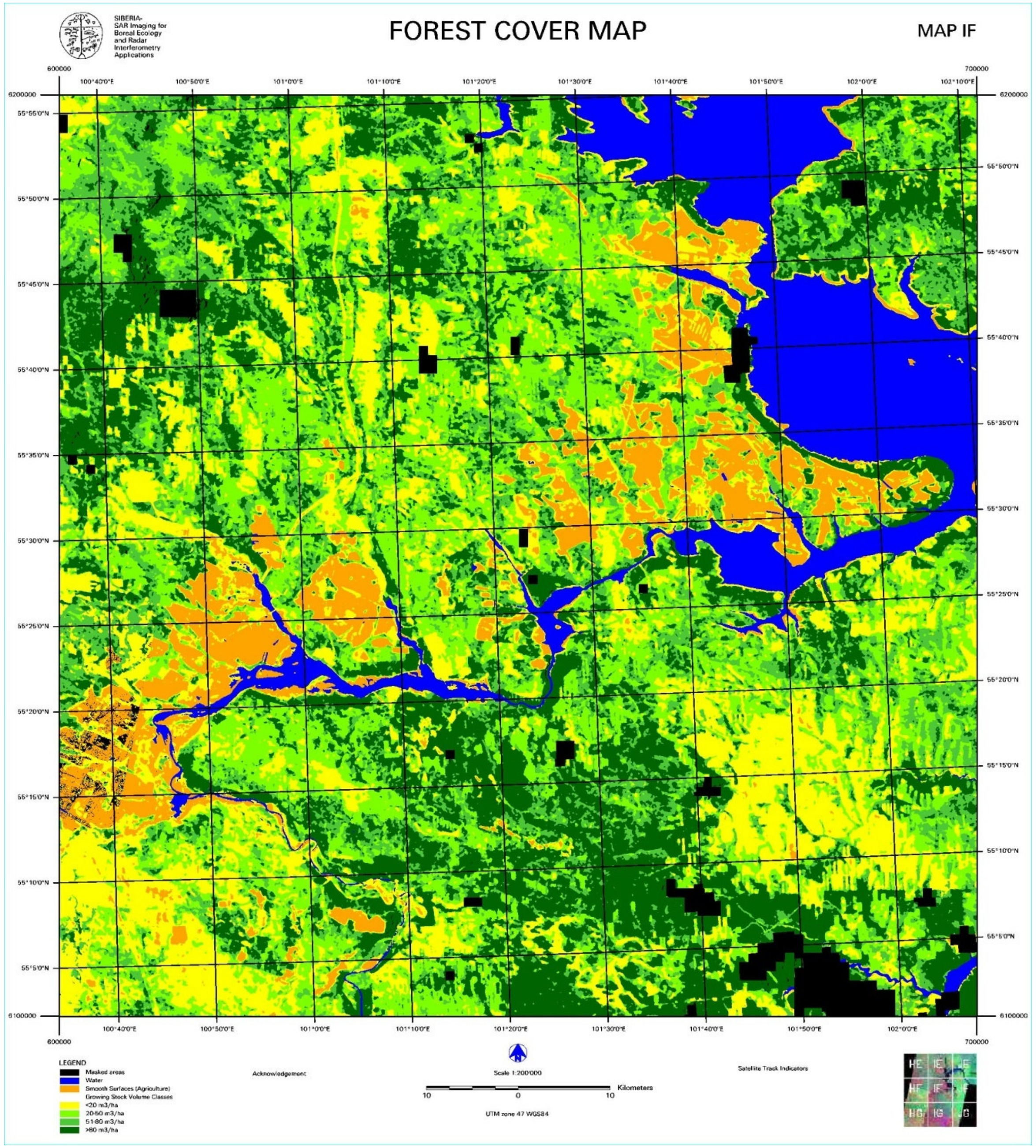
ALOS: 44 days.

Coherence image



80-pixel Tandem coherence in Bratsk region, Siberia.
ERS-1: 23/9/97
ERS-2: 24/9/97

Biomass from ERS interferometry and JERS L-band data



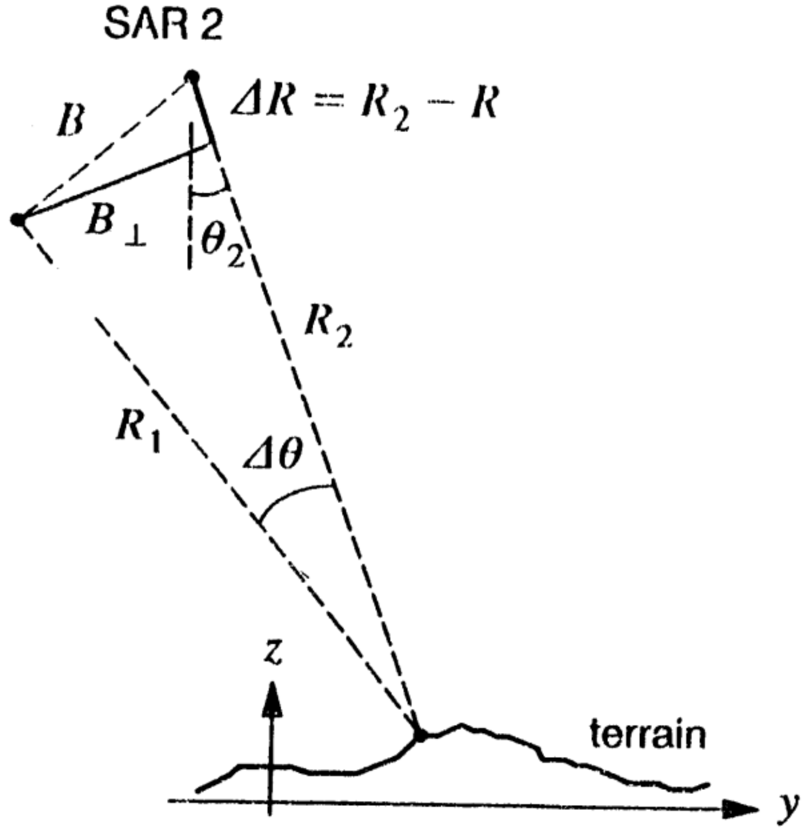
SIBERIA (SAR Imaging for Boreal Ecology and Radar Interferometry Applications), EU/CEO project no ENV4-CT97-0743.

ACHIEVEMENT: *Generation of a forest map of central Siberia* from ERS-1/ERS-2 Tandem coverage 1997
JERS coverage 1998

RESULTS*:* 96 forest cover maps (100 km X 100 km) covering 650000 km2. An example of map is shown on the left.

🟦	Water
🟧	Agriculture, bogs, grass
🟨	Open forest, cut, fire
🟩	Forest 10-30 t/ha
🟩	Forest 30-50 t/ha
🟩	Forest> 50 t/ha

SAR Interferometry



Height sensitivity =

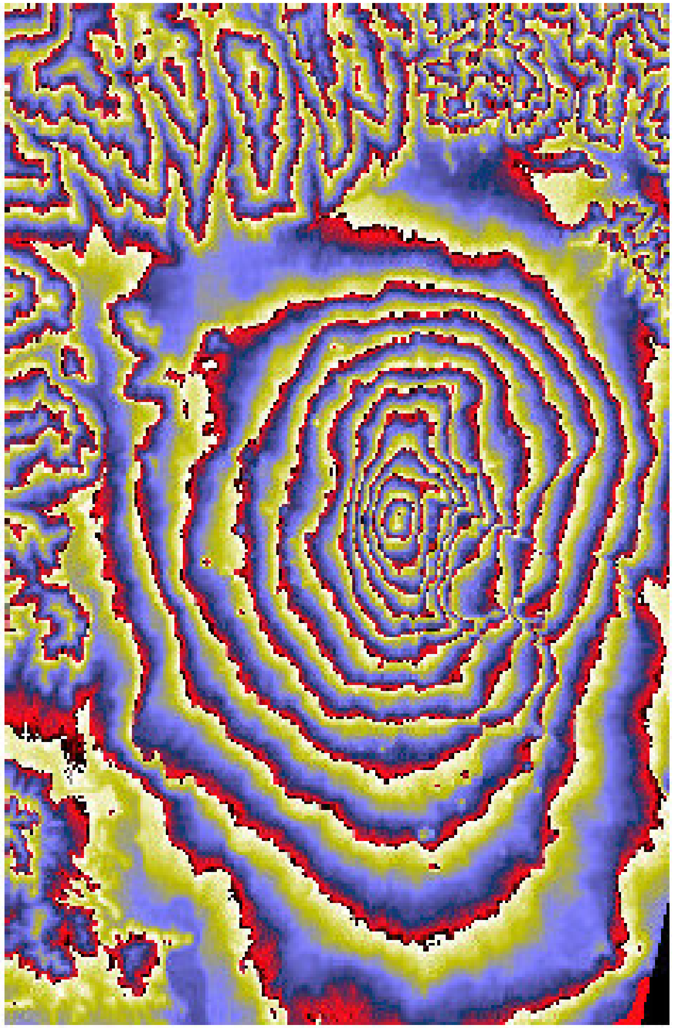
$$\frac{\partial \phi}{\partial z} = \frac{4\pi}{\lambda} \frac{B_{perp}}{R \sin \theta}$$

where φ is the interferometric phase, which is the phase of the complex correlation of the two images:

$$\phi = \mathrm{Arg}\langle \rho \rangle \text{ where } \rho = \frac{\langle s_1 s_2^* \rangle}{\sqrt{\langle |s_1|^2 \rangle \langle |s_2|^2 \rangle}}$$

Geometric information in interferograms

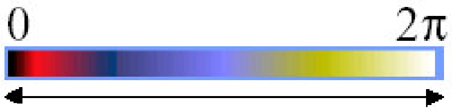
DEM



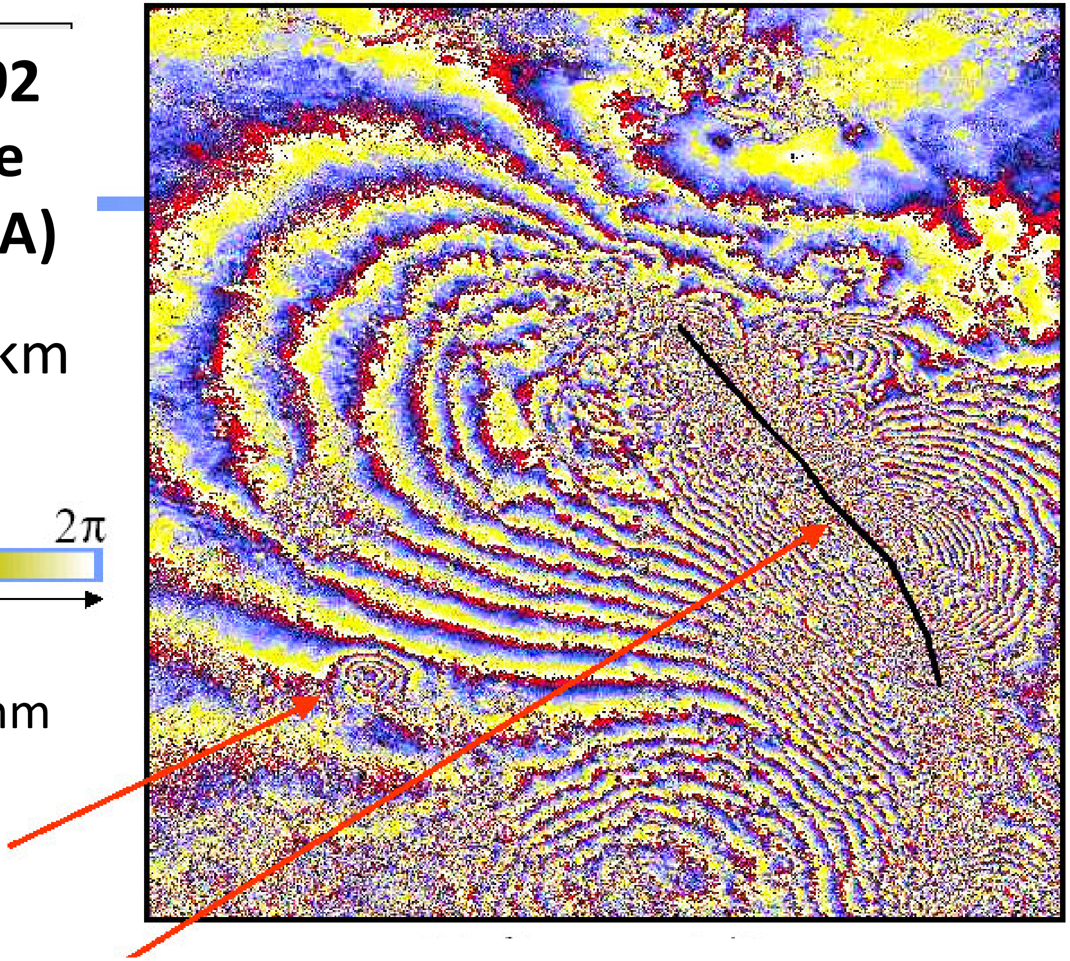
Differential InSAR

28 June 1992
Earthquake
Landers (USA)

90 km x 90 km



One fringe:
displacement of 28 mm



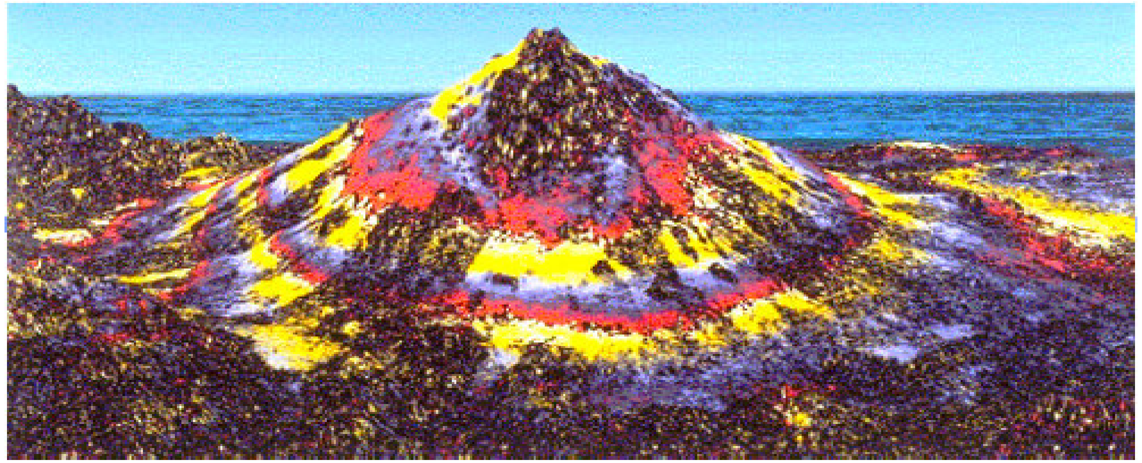
Post seismic
deplacement

Main fault

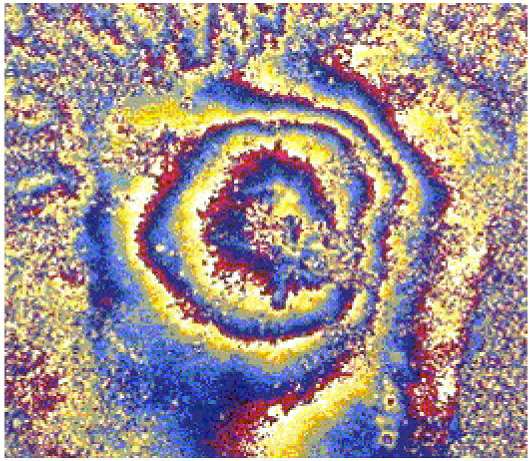
Interferogram from ERS-1
April 1992 and June 1993

InSAR applications

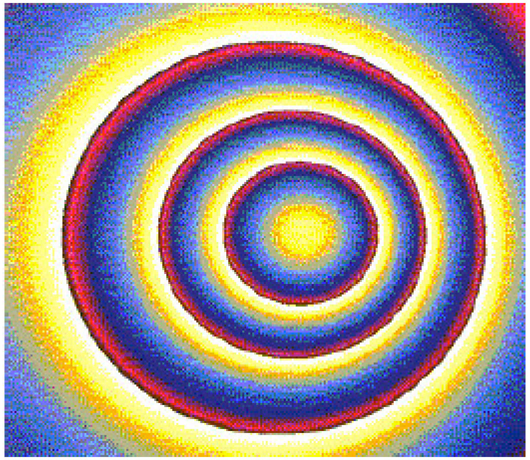
Volcano monitoring



30 ERS images
05-92 to 10-93
Etna

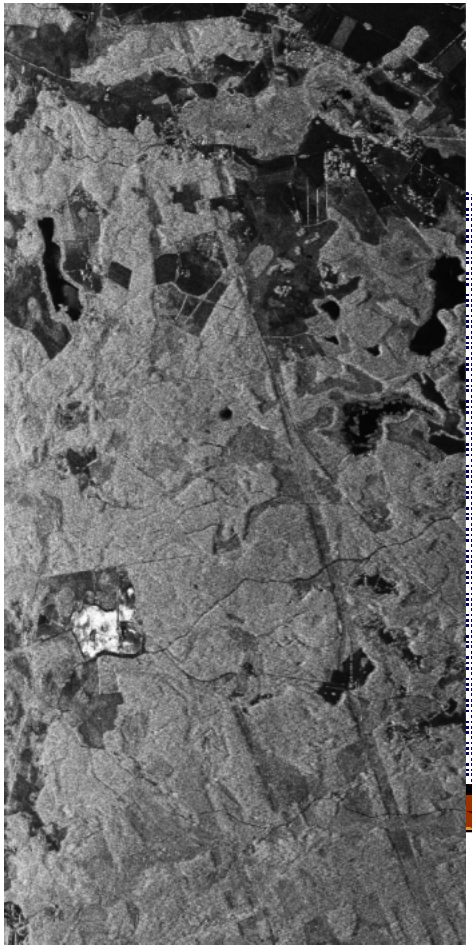


Interferogram

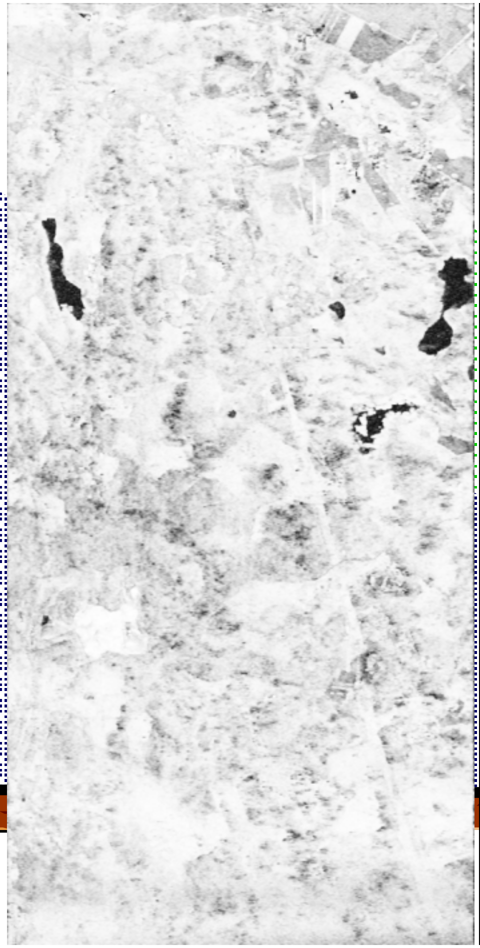


Modelling of the deflation (IPG)

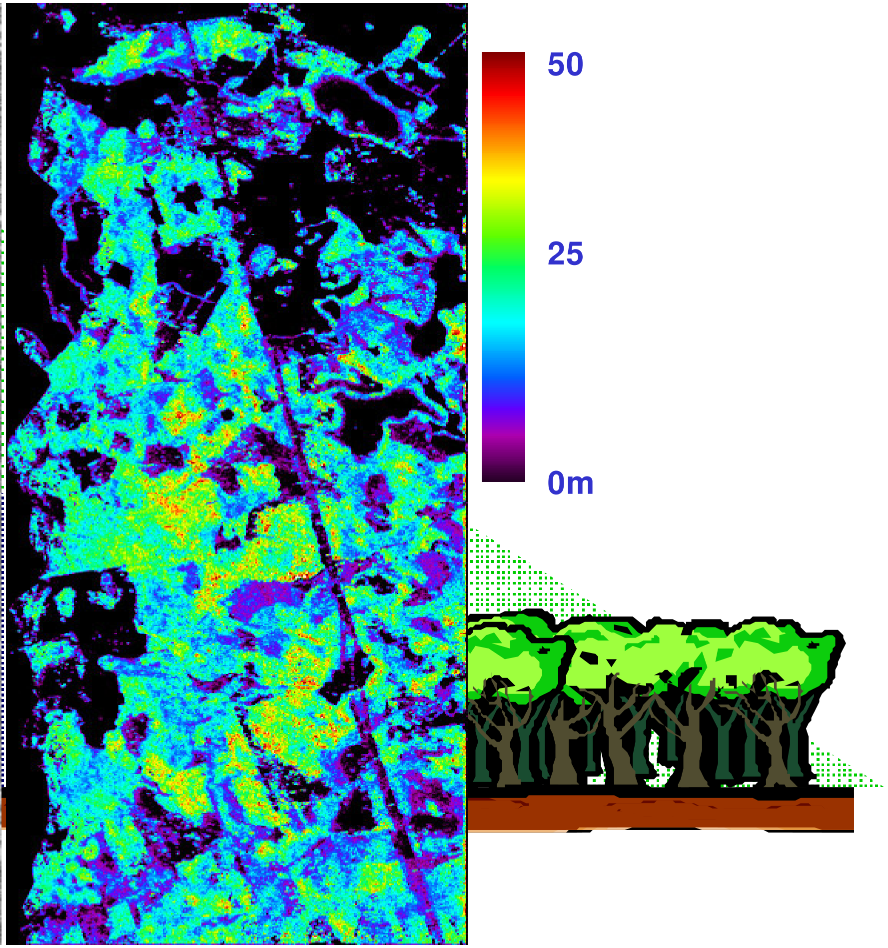
Forest height from Polarimetric InSAR



Amplitude Image L- HH



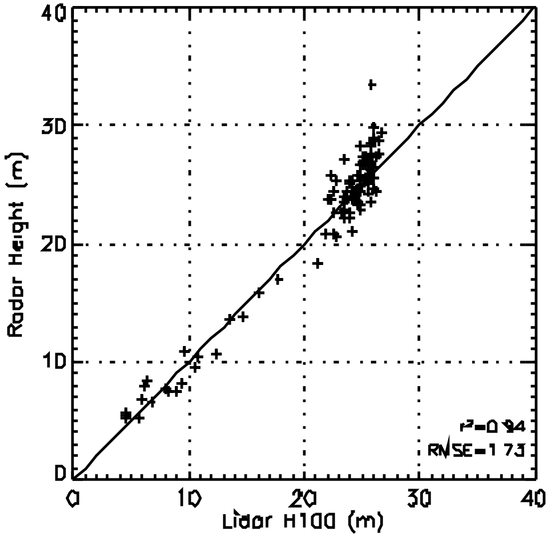
Volume Coherence



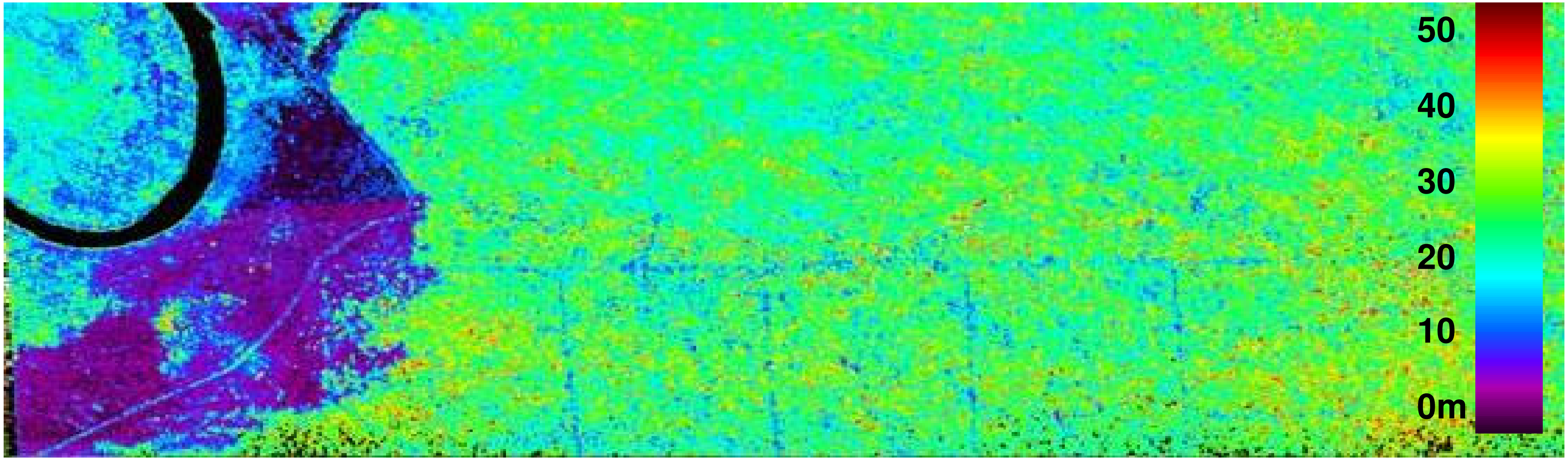
Forest Height Map

Forest height from Pol-InSAR

P-band height measurement verified during the ESA INDREX-II airborne campaign in Indonesia



Mawas, Indonesia



Summary

The basic types of SAR measurements are:

1. The complex backscattering coefficient
2. The complex correlation coefficient, phase difference and coherence: crucial for polarimetry and interferometry

The application determines the frequency selection and the need for polarisation. These each respond to different physical properties of the scene.

A critical concept is the Equivalent Number of Looks (ENL). This is crucial in knowing how many independent measurements we need to average to achieve an application – this determines the usable resolution.